LOW LEVEL R.F. BUILDING BLOCKS

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ABSTRACT
The low level R.F. system generates the signals driving the high power equipment, and processes the information coming back from the various beam and hardware P.U.s. The methods used to build the necessary low level functions are derived from the telecommunication and radar domains. The basic principles are described, and practical realizations are sketched using today's technology. An attempt is also made to indicate where the future evolution of electronics might lead.

1. INTRODUCTION

A number of contributions in the proceedings of this school depict and explain the operation of low level R.F. systems. The aim of the present paper is to describe some of the fundamental low level functions. Advancement of electronics technology makes it a fast changing domain (contrarily to high power installations) where any detailed hardware description can be quickly obsolete. Consequently the following lecture :

- tries to gather information relevant to accelerator design, previously disseminated into numerous textbooks dealing with communications and radar technology.
- concentrates on the principles,
- suggests practical realization without pretending to exhaust the list of hardware alternatives,
- indicates where modern digital technology can bring noticeable advantages.

For signal generation, the principles of Phase Locked Loops (P.L.L.), Direct Digital Synthesis (D.D.S.) and mixing are treated.

For signal processing also, the frequency conversion method (mixing or heterodyning) is described, and extensively applied. The principle of the superheterodyne is presented. Transmission of modulation through a linear system is analysed and modulation/demodulation techniques are explained. Finally filters, phase shifters and delays are also covered.

2. R.F. SYNTHESIS TECHNIQUES

2.1 Phase Locked Loops (P.L.L.)

Phase Locked Loops are fundamental building blocks for all kinds of R.F. equipment, and an impressive list of textbooks is available (Refs. [1] to [3] for instance). Their widespread use has been triggered by the availability of sophisticated components in the form of cheap integrated circuits. Principles of operation and common terminology are described below. More complex applications are examined in later sections, associating the various synthesis techniques.

2.1.1 Basic system

Lock acquisition will not be treated, and the interested reader is invited to investigate this specialized subject in the literature (Ref. [1] for instance). When locking has been achieved the P.L.L. maintains phase tracking between the reference input and some output signal under its control. Figure 1 shows the simplest block diagram of a phase locked loop.

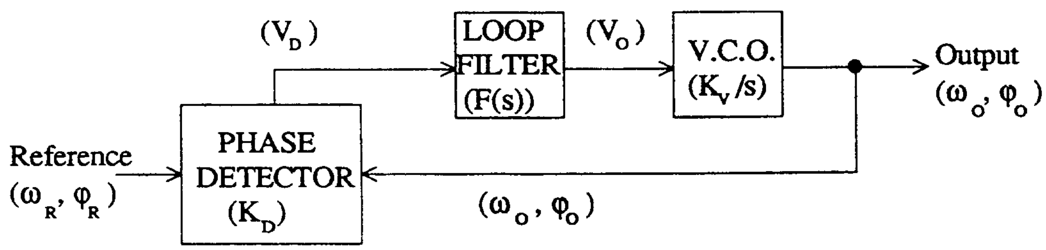


Fig. 1 Elementary block diagram of a P.L.L.

The fundamental components of a P.L.L. and their transfer functions are :

- the Phase Detector, which is ideally linear : $V_D = K_D(\varphi_R - \varphi_O)$ (1)
- the Loop Filter which determines the loop dynamic behaviour : F(s)
- the Voltage Controlled Oscillator (V.C.O.) whose output angular frequency ω_O (in rad/s) is proportional to its control voltage V_O : $\omega_O = K_V V_O$. (2)
  Angular frequency being the derivative of phase : $\varphi_O = \omega_O/s$ (3)
  and the transfer function of the V.C.O. is : $\varphi_O/V_O = K_V/s$. (4)

The closed loop transfer function for the phase of the reference φ_R to the phase of the loop output φ_O is given by :

$$\frac{\varphi_O}{\varphi_R} = \frac{K_D K_V F(s)}{s + K_D K_V F(s)} \,. \tag{5}$$

If F(s) = constant, the P.L.L. is a first-order low-pass filter for phase modulation.

Usually :

$$F(s) = K\left(\frac{s+\omega_1}{s}\right) \tag{6}$$

and the loop is of type 2 (2 integrators) and second order (denominator's degree in the transfer function). The closed loop transfer function is then :

$$\frac{\varphi_O}{\varphi_R} = KK_D K_V \left(\frac{s+\omega_1}{s^2 + sKK_D K_V + KK_D K_V \omega_1}\right) \tag{7}$$

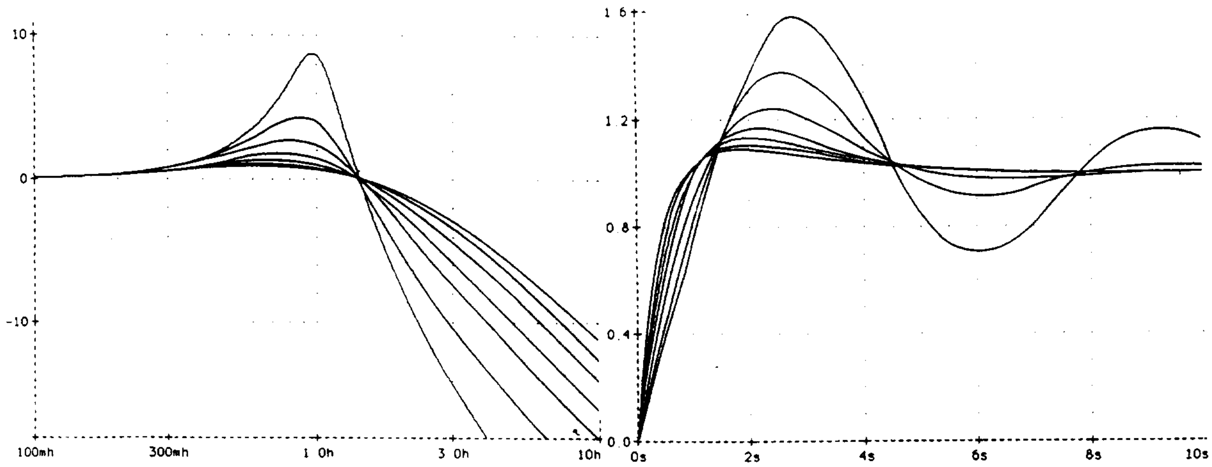
which can be conveniently written as :

$$\frac{\varphi_O}{\varphi_R} = 2\varsigma\omega_n \left(\frac{s + \frac{\omega_n}{2\varsigma}}{s^2 + 2\varsigma\omega_n s + \omega_n^2}\right) \tag{8}$$

using the damping ratio $\varsigma = \frac{1}{2}\sqrt{\frac{KK_D K_V}{\omega_1}}$ (9)

and $$\omega_n = \sqrt{KK_DK_V\omega_1}. \tag{10}$$

Figure 2(a) illustrates the frequency response corresponding to Eq. (8) when $\omega_n = 2\pi$ rad/s (1 Hz) and for a damping ratio ς between 0.2 and 1.4, in steps of 0.2 . The oscillatory behaviour observed for $\varsigma \leq \sqrt{2}$ (critical damping) is also visible in the time domain output for a unit step perturbation at the input (Fig. 2(b)).



(a) (b)

Fig. 2 Frequency and unit step responses of second order P.L.L.
(damping ratio from 0.2 to 1.4)

2.1.2 Phase Detector

The phase detector can be of the analogue or digital type. A high rejection of the input frequency is mandatory to avoid modulation of the V.C.O. and degradation of the output spectrum. The Sample & Hold detector has, by construction, a high rejection of the sampling (reference) frequency. The other detectors need a low-pass filter with a cut-off frequency $\omega_{LP}/2\pi$ much lower than the smallest input frequency $\omega_{Rmin}/2\pi$ (typically $\omega_{LP} < \omega_{Rmin}/10$).The commonly used elements are listed in Tables 1 and 2, with their phase detection characteristics.

The analogue phase detector is a multiplier, whose output characteristic is sinusoidal with sinusoidal input signals, or linear with square waves. Real analogue multipliers can be used from quasi-DC to 100 MHz (variable transconductance amplifiers,...). Above this range the double balanced mixer is the ubiquitous solution (see Section 3 for detailed description).

The simplest digital phase detector is built with an exclusive OR gate. It shows a linear range of π rad. Its major limitation stems from the need for a well defined duty cycle on both inputs. Frequency division by 2 is sometimes introduced in front of the gate to provide well defined square waves (for instance in the Analog Devices part AD9901, Ref. [4]).

The Sample & Hold is well matched to low frequency operation, thanks to its built-in rejection of the input frequencies due to the sampling process.

Table 1
Conventional Phase Detector circuits

<table>
<tr><th>CIRCUIT</th><th>CHARACTERISTICS</th></tr>
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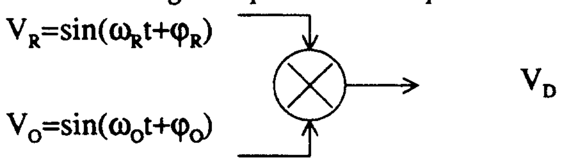
Analogue 4 quadrant multiplier



$$V_D=\frac{1}{2}\left\{\begin{array}{l}\cos[(\omega_R-\omega_O)t+(\varphi_R-\varphi_O)]-\\ \cos[(\omega_R+\omega_O)t+(\varphi_R+\varphi_O)]\end{array}\right\}$$

$\Downarrow$ (low pass filtering)

$$\overline{V}_D=\frac{\cos(\varphi_R-\varphi_O)}{2}$$

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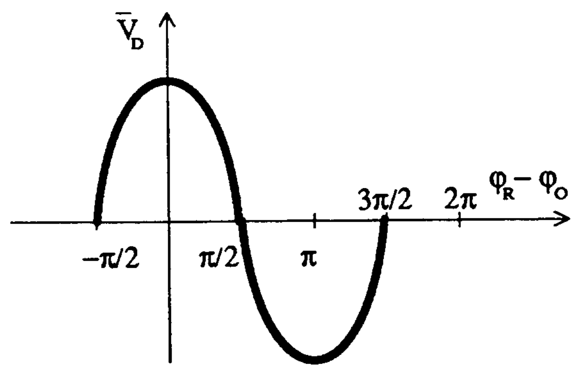

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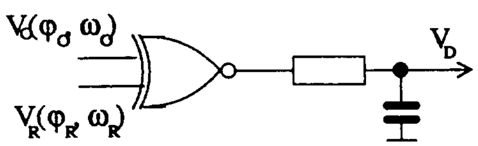
Digital Exclusive OR
or square wave driven analogue multiplier



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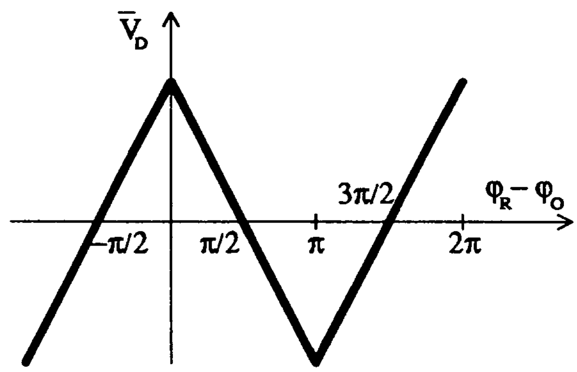

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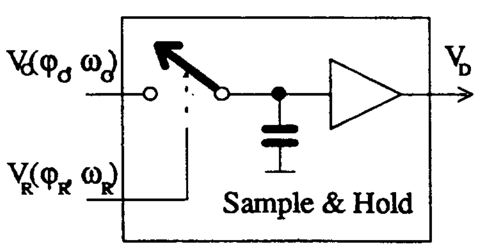
Sample & Hold Detector



(Sampling happens at the zero crossing with positive slope of V_R)

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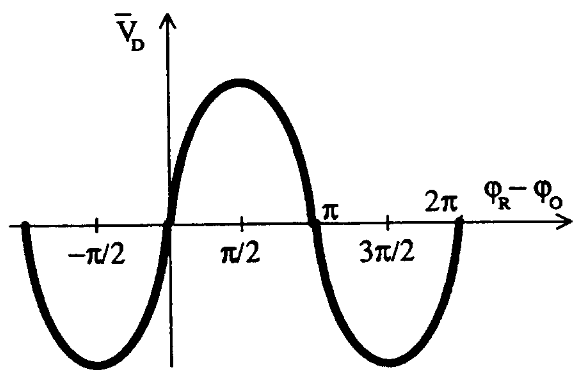

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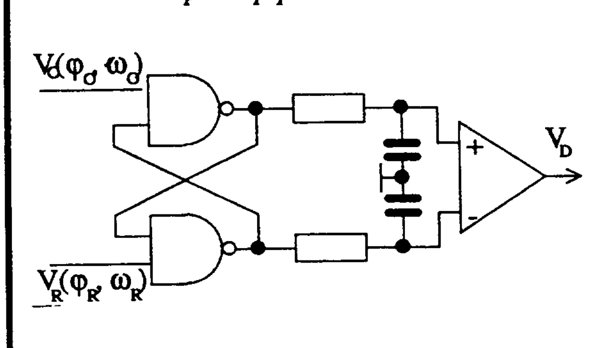
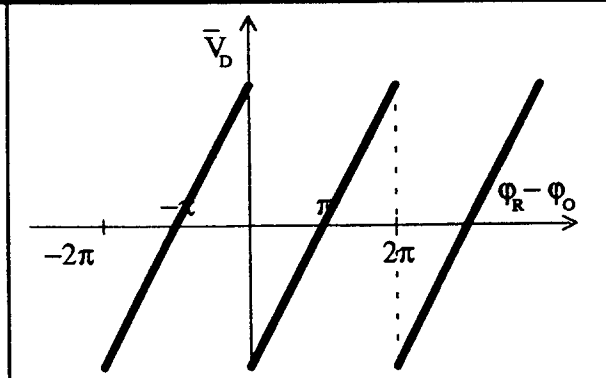
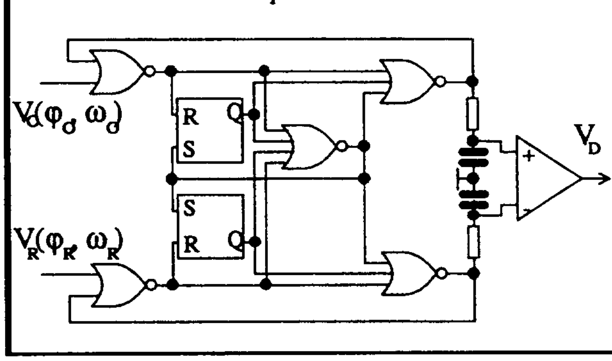
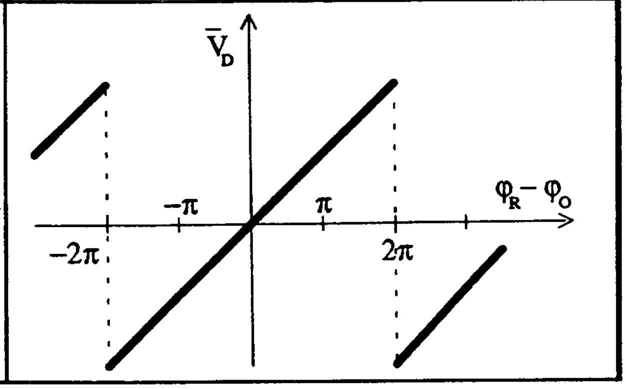
However the most used types of phase detectors are listed in Table 2. They exhibit a linear characteristic over 2π or 4π rad, while convenient to implement up to a few tens of MHz, when built with high speed logic components (ECL).

Special integrated circuits perform the complete "Tri-state phase detector" operation (for instance Motorola part MC12040, Ref.[5]).

Table 2
Edge triggered Phase Detectors

<table>
<tr><th>CIRCUIT</th><th>CHARACTERISTICS</th></tr>
<tr><td>Flip-Flop phase detector

</td><td>
</td></tr>
<tr><td>Tri-state phase detector

</td><td>
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2.1.3 Loop Filter

Operational amplifiers are used to build active filters. A typical realization of the filter involved in a type 2 second order loop is shown in Fig. 3.

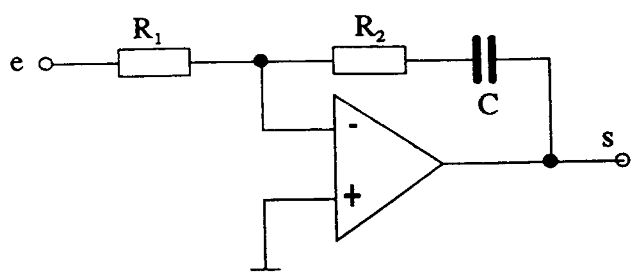


Fig. 3 Active filter for a second order loop

Its transfer function is : $$F(s) = \frac{s}{e} = K\left(\frac{s+\omega_1}{s}\right) \tag{11}$$

$$\text{with } K = \frac{R_2}{R_1} \text{ and } \omega_1 = \frac{1}{R_2 C}$$

The corresponding Bode plot for the open loop gain of the complete loop is given in Fig. 4.

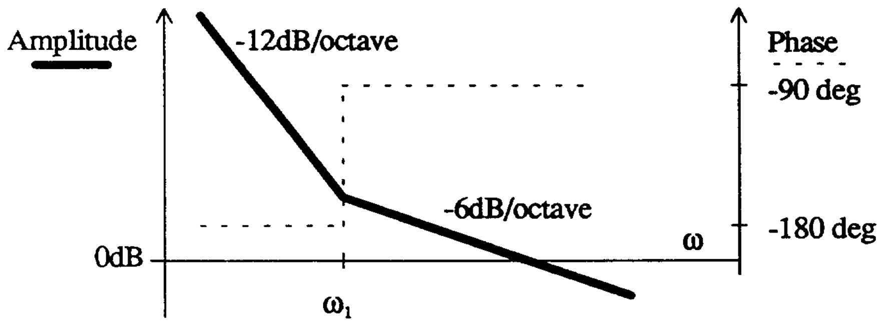


Fig. 4 Open loop gain of a second order, type 2 loop

More complicated transfer function can help further optimize the overall loop performance (Refs. [1] to [3]).

2.1.4 Voltage Controlled Oscillator (V.C.O.)

Two types of oscillators can be distinguished:

- the multivibrator, which has a rather linear frequency to voltage (or current) characteristic but a square-wave output.
- the varactor tuned oscillator which has a good quality sine-wave output, but a rather non-linear relation between frequency and voltage .

Figure 5 shows a simplified schematic of a typical astable multivibrator. The capacitor is alternately charged with one polarity until a threshold is attained and then with the reverse polarity.

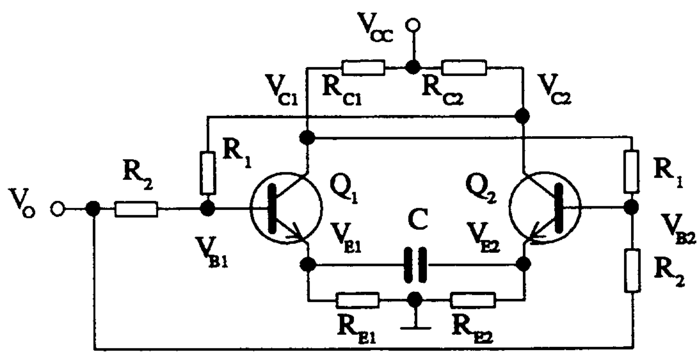


Fig. 5 Simplified schematic of an astable multivibrator type of V.C.O.

Let us assume that transistor Q_1 is conducting and saturated, and that C is not charged. Then V_{C1} is low and Q_2 is blocked. V_{C2} is at V_{CC} which drives Q_1 into conduction. Our initial assumption is validated. The current flowing through Q_1 is shared between R_{E1} and $C+R_{E2}$, loading the capacitor C. Since C is initially not charged, the current is equally distributed to R_{E1} and R_{E2}, providing an equal voltage on the emitters of Q_1 and Q_2. As C charges, V_{E2} decreases because of the decreasing current through R_{E2}. When V_{B2}-V_{E2} approaches 0.7 V , Q_2 becomes conducting, which drives V_{C2} lower and reduces conduction of Q_1. V_{E1} is then lower, which amplifies the process and the system switches to a state where Q_1 is blocked and Q_2 is saturated. Current now flows into C in the opposite direction.

Toggling results, at a rate determined by the voltage V_O. In practical application the frequency is made a linear function of V_O, using current sources instead of the resistors R_{E1} and R_{E2} [6].

Filtering is required to obtain a sine-wave from the square-wave output available on the collectors.

In a varactor tuned oscillator a variable capacitance diode is used to control the centre frequency of a resonator. A typical implementation taken from Motorola documentation is shown in Fig. 6. A sine-wave can directly be obtained thanks to the filtering of the resonator.

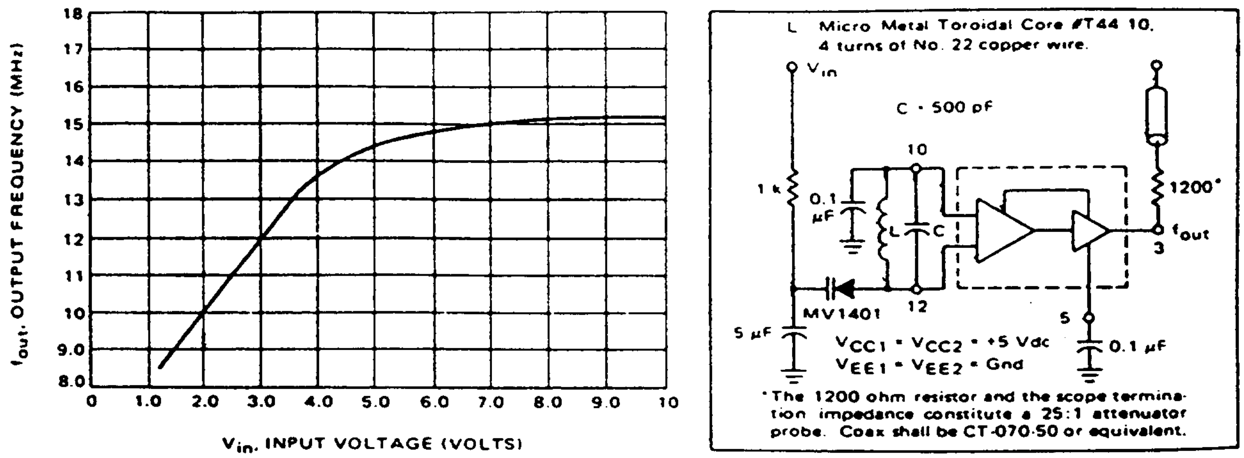


Fig. 6 Varactor tuned V.C.O.

2.1.5 Practical system

As illustrated in the block diagram of Fig. 7, showing a practical P.L.L., a programmable divider is commonly inserted between V.C.O. and phase detector for a numerical control of the output frequency. The open loop gain varies like 1/N, and compensation can be necessary inside the loop, when N covers a large range.

The output angular frequency is : $\omega_O = N\omega_R$. (12)

Since N is an integer, the frequency step size is ω_R. Consequently, ω_R becomes small when one aims at a high resolution, with the net result that the loop bandwidth gets even smaller (see Section 2.1.2). The "Digiphase" principle [7] and the "Fractional N division technique" [8] have been developed to circumvent this limitation, but their complication limits their use to commercial instruments.

Two analogue control inputs are also implemented in Fig. 7 for:

- low frequency phase modulation (inside the loop bandwidth) : useful for application as phase shifter,
- AC frequency modulation (outside the loop bandwidth).

The control voltage for the V.C.O. (V_O) governs the frequency modulation of the oscillator. It is a convenient signal to monitor, whose quality is a consequence of the linearity of the V.C.O..

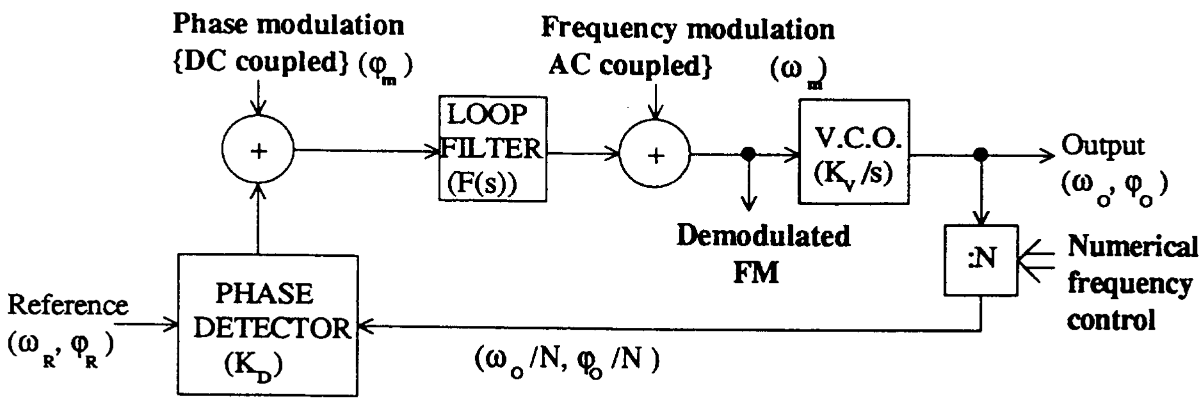


Fig. 7 Block diagram of a practical P.L.L.

2.1.6 Applications of P.L.L.s

P.L.L.s are commonly utilized for :

- accurate and low noise frequency sources (inside the loop bandwidth the power density of noise is a copy of the reference one, multiplied by the frequency multiplication factor N),
- filtering of the phase noise on the reference signal (using small loop bandwidth, so that even near the carrier the noise power density is that of the free-running V.C.O.),
- phase modulation/phase shifting (where linearity and dynamic range are defined by the phase detector),
- frequency (phase) demodulation of the reference signal.

2.2 Direct Digital Synthesis (D.D.S.)

2.2.1 Principle

Direct Digital Synthesis up to a few tens of MHz has been made possible by the advances in electronics technology [2, 9]. Based on digital arithmetic circuits and Digital to Analogue Converter (D.A.C.) its basic block diagram is shown in Fig. 8.

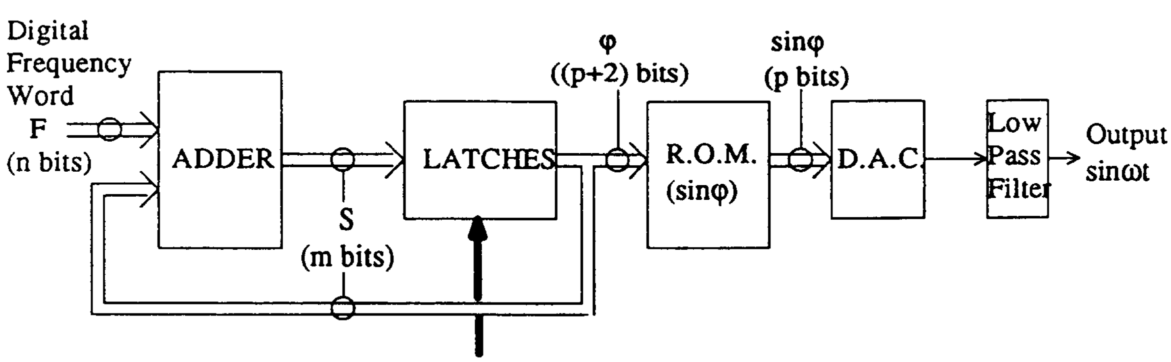


Fig. 8 Elementary block diagram for D.D.S.

At each clock pulse the content S of the Accumulator (output of the latches) is incremented by F. Since the accumulator regularly overflows, S is a quantized sawtooth (if converted to analogue). Transforming a truncated subset φ of S (the p+2 most significant bits of S) into sinφ provides a continuous sine-wave after digital to analogue conversion (Fig. 9).

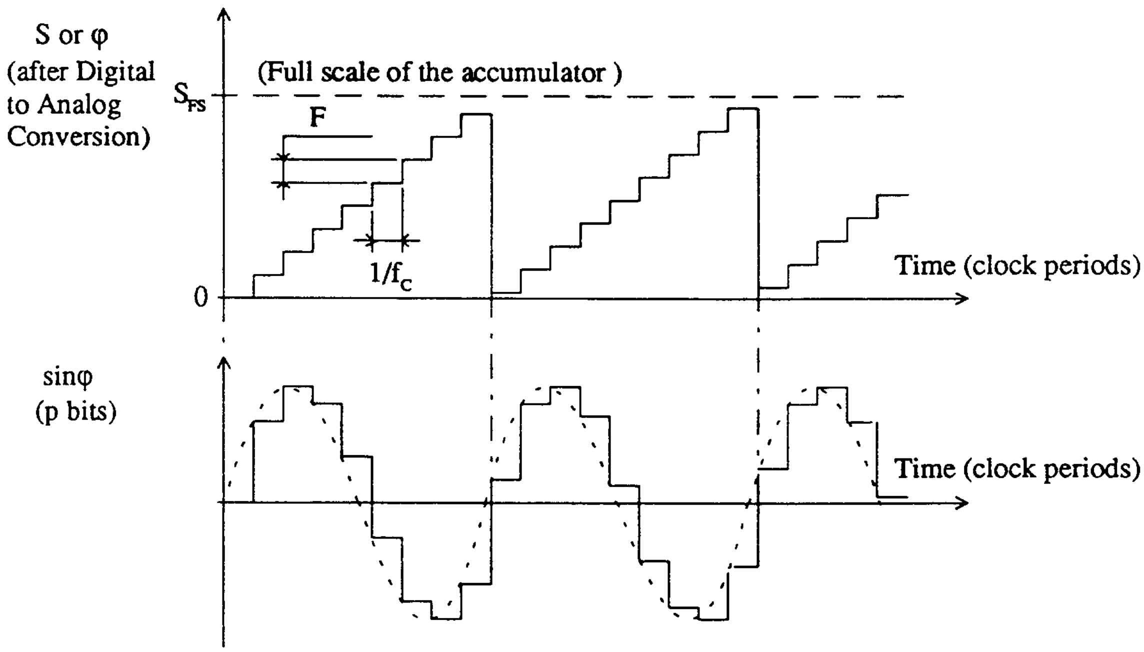


Fig. 9 Signal generation in D.D.S.

Beginning with S = 0 at time t = 0, after n clock periods : S = nF. The output sine-wave will complete a full period after a duration T given by :

$$\boxed{T = \frac{S_{FS}}{F} \cdot \frac{1}{f_C} \quad \Rightarrow \quad f = \frac{1}{T} = \frac{F}{S_{FS}} \cdot f_C} \tag{13}$$

with:

f : frequency of output wave (Hz)
T : period of output wave (s)
F : digital input control word
f_C : clock frequency
S_{FS} : full scale value of the accumulator

Low pass filtering is needed to smooth the quantized sine-wave (in the time domain) or to eliminate the spurious frequency components (in the frequency domain).

2.2.2 Performance

Three quantization effects limit performance :

i) *Frequency resolution.*

Frequency resolution increases with the numbers of bits n and m processed in the accumulator. Typical figures for today's technology are given in Table 3.

Table 3
Commercial D.D.S. parts

Component [Company/part no.]	Maximum clock frequency (f_C)	Number of bits for F (n)	Number of bits for output (p)
Plessey SP2001	350 MHz	16	8
Stanford Telecom STEL-2173	1000 MHz	32	8
Sciteq ADS-2-101	400 MHz	25	8
Analog Devices AD9950	300 MHz	32	10 (no internal look-up table)

ii) *Amplitude quantization.*

Because of the limited number of bits used by the D.A.C., the output signal can be considered as the sum of a pure sine-wave and a waveform corresponding to the amplitude quantization error. Each spurious frequency component of that error signal is expected in practice not to exceed $1/2^p$ of the main sine-wave, or -6p dBc (dB with respect to the carrier) (Table 4).

Table 4
Level of amplitude quantization spurious

p (Number of bits for the D.A.C.)	Maximum spurious level with respect to carrier (in dBc)
8	-48
10	-60
12	-72

iii) *Time quantization.*

The unfiltered output is a quantized sine-wave, sampled and held at the clock rate f_C. Its spectrum contains spurious lines above the Nyquist frequency $f_C/2$. The output low-pass filter is necessary to clean the signal (see Fig. 10). To facilitate construction of the filter, the maximum output frequency is generally limited to $f_C/4$.

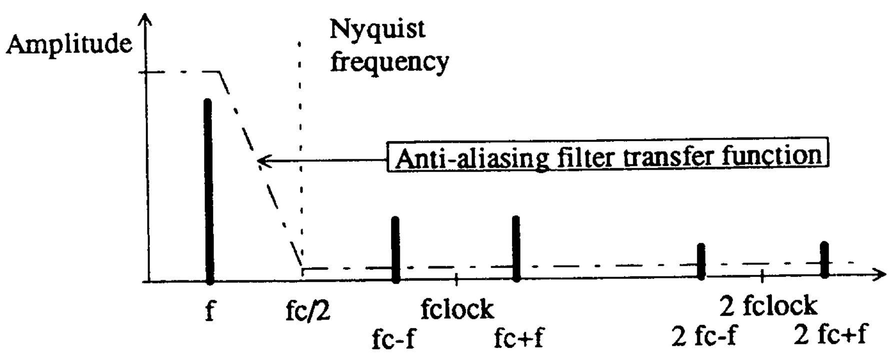


Fig. 10 D.D.S. frequency spectrum and output filter characteristic

The most interesting features of Direct Digital Synthesis are:

- high accuracy and stability (depending only on clock performance),
- very fast response time (a few clock periods + the output filter delay),
- perfect phase continuity while changing frequency,
- good phase noise performance (function of clock source),
- easy simultaneous generation of sine and cosine waves (often used in real synthesizers as described later in Section 3.3).

2.2.3 Practical system

Resetting of the accumulator allows a precise control of phase evolution as a function of time. This feature is very interesting for sophisticated synchronization schemes between accelerators.

Digital phase modulation is possible with an adder in the path of φ. Similarly, digital amplitude modulation only requires a multiplier in the path of $\sin\varphi$, before the D.A.C..

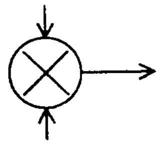
Designs with discrete components (M.S.I.) can nowadays work up to roughly 10 MHz (40 MHz clock), using ECL parts. Higher frequencies are the domain of special integrated circuits and hybrid modules (Examples are given in Table 3).

3. MIXING AND HETERODYNING

3.1 Principle

Any non-linear device generates intermodulation products when subject to signals of different frequencies. The most commonly used function for this purpose is multiplication. Figure 11 shows its operation.

$V_R = v_R\cos(\omega_R t+\theta_R)$



$$V_{out} = V_R V_L = v_R v_L \cos(\omega_R t+\theta_R)\cos(\omega_L t+\theta_L) \qquad (14)$$

$V_L = v_L\cos(\omega_L t+\theta_L)$

Fig. 11 Multiplier operation

V_{out} can be expressed in the form of a sum of two cosine-waves :

$$V_{out} = \left(\frac{v_R v_L}{2}\right)\left[\cos\left((\omega_R-\omega_L)t+(\theta_R-\theta_L)\right)+\cos\left((\omega_R+\omega_L)t+(\theta_R+\theta_L)\right)\right] \qquad (15)$$

After Fourier transformation, the results in the frequency domain are illustrated in Fig. 12, where the phases of the various complex components are given according to the following expression :

$$V_{out} = \left(\frac{v_R v_L}{2}\right)\left\{\begin{array}{l} \frac{1}{2}\left[e^{j(\omega_R-\omega_L)t+(\theta_R-\theta_L)}+e^{-j(\omega_R-\omega_L)t-(\theta_R-\theta_L)}\right] \\ +\frac{1}{2}\left[e^{j(\omega_R+\omega_L)t+(\theta_R+\theta_L)}+e^{-j(\omega_R+\omega_L)t-(\theta_R+\theta_L)}\right] \end{array}\right\} \qquad (16)$$

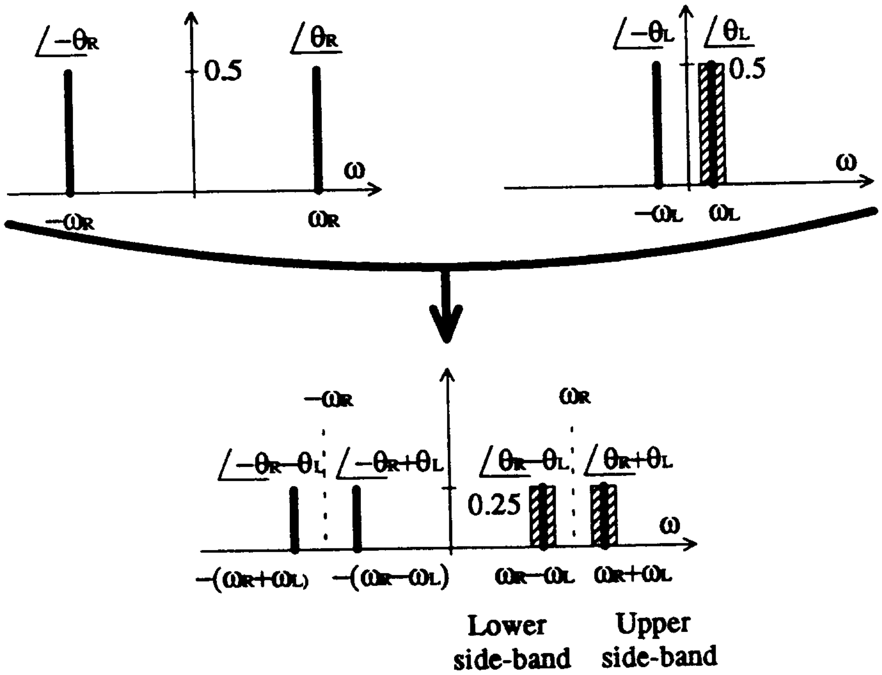


Fig. 12 Frequency spectrum after mixing in an ideal multiplier

Analogue multipliers can be employed below a few tens of MHz (see Section 2.1.2), and other types of circuits may present some interesting advantages [6] in special applications. However double balanced mixers are the most often used components for this function, thanks to their advantages.The schematic of a double balanced mixer is shown in Fig. 13 (a), and its operation is described in Figs. 13 (b) and (c).

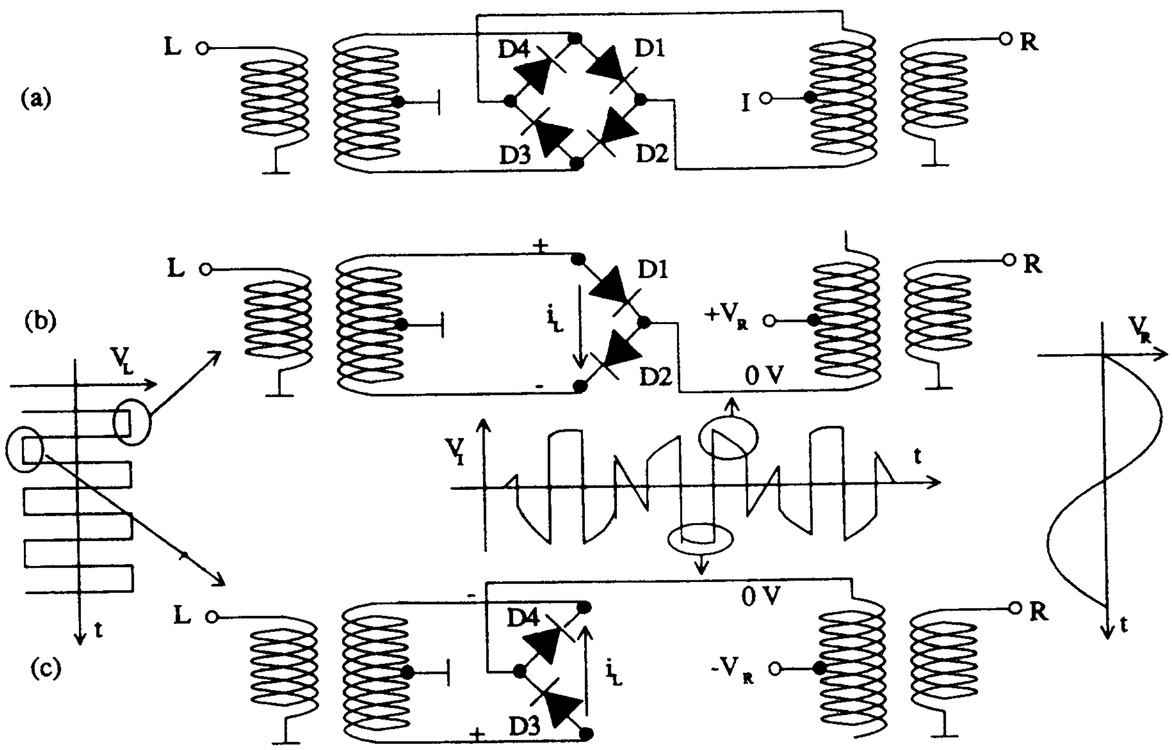


Fig. 13 Double balanced mixer schematic and operation

The Local Oscillator port L is driven by a square-wave which switches on and off each pair of diodes. When D1+D2 are conducting and D3+D4 are blocked (Fig. 13 (b)), the bottom end of the transformer driving the Intermediate Frequency port I is grounded. The output voltage (I being terminated into the nominal impedance) is then + V_R. When D3+D4 are conducting and D1+D2 are blocked, the upper end of the transformer is grounded and -V_R is provided at the I port. The mixer makes the product of the R.F. signal V_R by the square-wave V_L. The frequency spectrum at the output results from the combination of the frequency at the R port with the various harmonics of the square-wave.

The most important characteristics of double balanced mixers are :

- isolation between ports (rejection of direct feedthrough from R to I, L->R, L->I),
- wide frequency range (commercial units are available from D.C. to tens of GHz, with up to 3 decades of bandwidth),
- low cost.

3.2 Basic Applications

3.2.1 Superheterodyne

The principle of the superheterodyne (Fig. 14) is to change the variable R.F. angular frequency ω_R of an input signal into a fixed angular frequency ω_I where signal processing takes place. At the cost of generating a local oscillator at ω_L, much is gained in ease of construction and in overall performance with respect to a system working directly at ω_R.

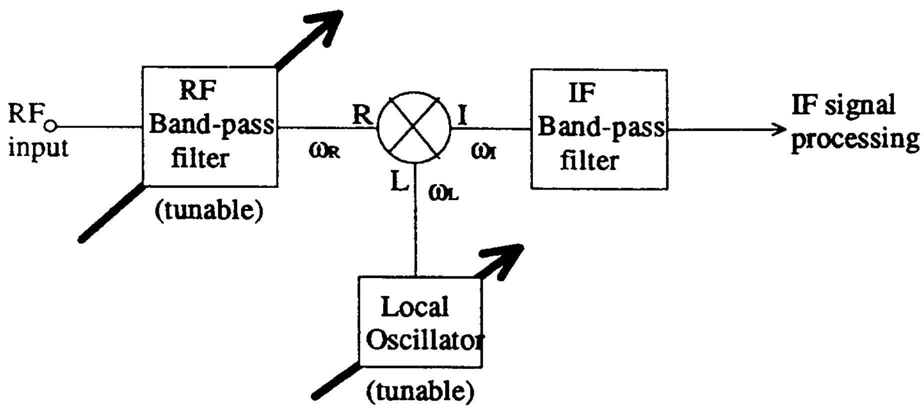


Fig. 14 Principle of Superheterodyne

Band-pass filtering at the Intermediate Frequency (I.F.) selects two frequency bands at the R.F. input. Angular frequencies close to $\omega'_R=\omega_L+\omega_I$ as well as $\omega''_R=\omega_L-\omega_I$ both generate an I.F. signal at ω_I, as illustrated in Fig. 15. A given system is designed to work for one of these bands. The other one is called the "**image**", and measures must be taken to avoid it perturbing the overall operation.

Band-pass filtering before mixing is a possible means to reject the image frequency. Another convenient means is provided by the device shown in Fig. 16, which is called an "Image rejection mixer". It provides the intermediate frequency signals corresponding to ω'_R and ω''_R on two distinct output ports. It employs a passive lossless element called a "Quadrature Hybrid", which can be made in various ways but is also available from a number of companies. The signal fed into any port is transmitted equally to two other ports, with the phasing indicated in Fig. 16. The fourth port is isolated.

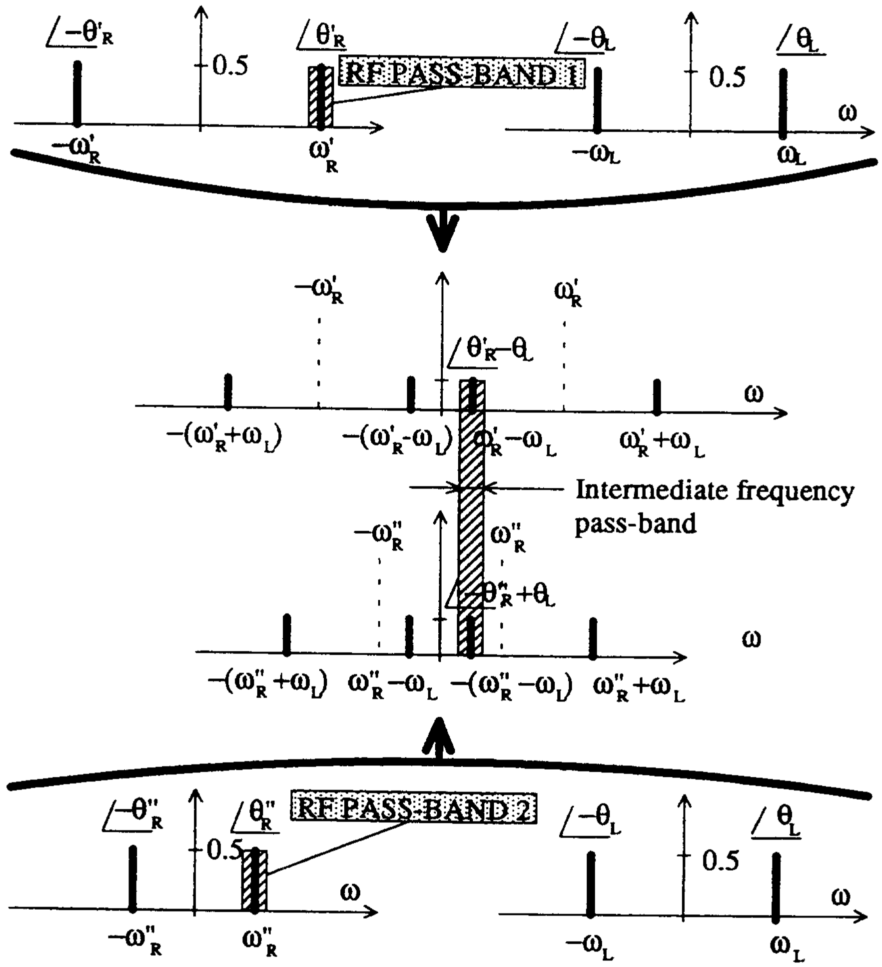
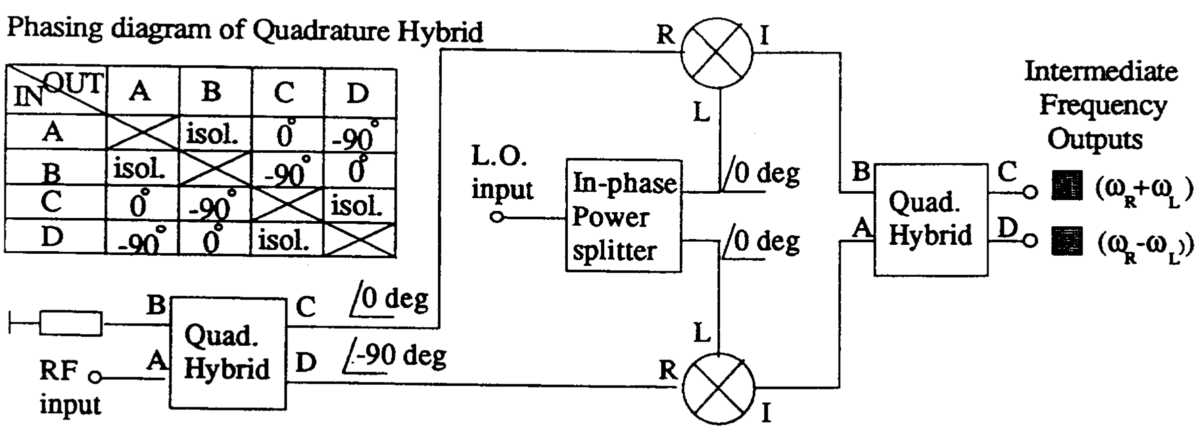


Fig. 15 Real and image frequency bands in a superheterodyne



IN \ OUT	A	B	C	D
A		isol.	0°	-90°
B	isol.		-90°	0°
C	0°	-90°		isol.
D	-90°	0°	isol.	

Fig. 16 Image rejection mixer

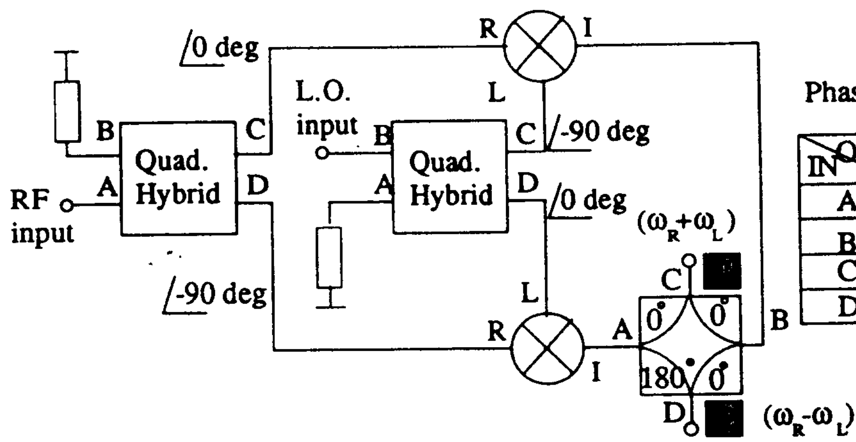
Its operation can be explained using trigonometric identities, or directly with the phase information given in Fig. 12. In Table 5 the relative phase shift for transmission from the input port to the output ports 1 and 2 is analysed. Addition or cancellation is then deduced in the various cases.

Table 5
Operation of an Image rejection mixer

Input angular frequency	I.F. channel	I.F. phase on port 1	I.F. phase on port 2	Transmission to port 1	Transmission to port 2
$\omega'_R=\omega_L+\omega_I$	Top	$\theta_R-\theta_L-\pi/2$	$\theta_R-\theta_L$	YES	NO
$\omega'_R=\omega_L+\omega_I$	Bottom	$\theta_R-\theta_L-\pi/2$	$\theta_R-\theta_L-\pi$	YES	NO
$\omega''_R=\omega_L-\omega_I$	Top	$-\theta_R+\theta_L-\pi/2$	$-\theta_R+\theta_L$	NO	YES
$\omega''_R=\omega_L-\omega_I$	Bottom	$-\theta_R+\theta_L+\pi/2$	$-\theta_R+\theta_L$	NO	YES

3.2.2 Signal generation

Mixing is also employed for signal generation. But even when fed with perfectly pure sine waves at ω_R and ω_L , an ideal mixer always generates two side-bands at $\omega_R+\omega_L$ and $\omega_R-\omega_L$ (Fig. 12). The unwanted band can be eliminated by filtering or using the "single side-band mixer" represented in Fig. 17. Apart from quadrature hybrids another type of passive lossless element called "Magic Tee" is used. The power entering one port is split in two equal halves, with the phasing given in this Figure. The fourth port is isolated. Operation of the "single side-band mixer" is detailed in Table 6, where the phase shifts through the system are analysed for the two output frequencies. It can be demonstrated that only the upper side-band is available at the output port 1, and only the lower side-band at the output port 2.



Phasing diagram of Magic Tee

IN \ OUT	A	B	C	D
A		isol.	0°	180°
B	isol.		0°	0°
C	0°	0°		isol.
D	180°	0°	isol.	

Fig. 17 Single side-band mixer

Table 6
Operation of the single side-band mixer

Output angular frequency	I.F. channel	I.F. phase on port 1	I.F. phase on port 2	Transmission to port 1	Transmission to port 2
$\omega_R+\omega_L$	Top	$\theta_R+\theta_L-\pi/2$	$\theta_R+\theta_L-\pi/2$	YES	NO
$\omega_R+\omega_L$	Bottom	$\theta_R+\theta_L-\pi/2$	$\theta_R-\theta_L+\pi/2$	YES	NO
$\omega_R-\omega_L$	Top	$\theta_R-\theta_L+\pi/2$	$\theta_R-\theta_L+\pi/2$	NO	YES
$\omega_R-\omega_L$	Bottom	$\theta_R-\theta_L-\pi/2$	$\theta_R-\theta_L+\pi/2$	NO	YES

3.3 Synthesizer examples

Mixing is often used to extend the maximum frequency of a given P.L.L. set-up, or simply to generate the Local Oscillator needed in a Superheterodyne receiver. Figure 18 illustrates such an application. The output angular frequency ω_O is mixed down with a signal at the intermediate angular frequency ω_{IF}, and the difference ω_O-ω_{IF} is divided by N before phase comparison with ω_R in the phase detector. When phase lock is achieved, one has :

$$\omega_R = \frac{\omega_O - \omega_{IF}}{N} \Rightarrow \omega_O = \omega_{IF} + N\omega_R. \tag{17}$$

If ω_R is the R.F. angular frequency and N=1, the loop generates the Local Oscillator frequency needed to heterodyne all R.F. signals to be processed to the constant intermediate angular frequency ω_{IF}.

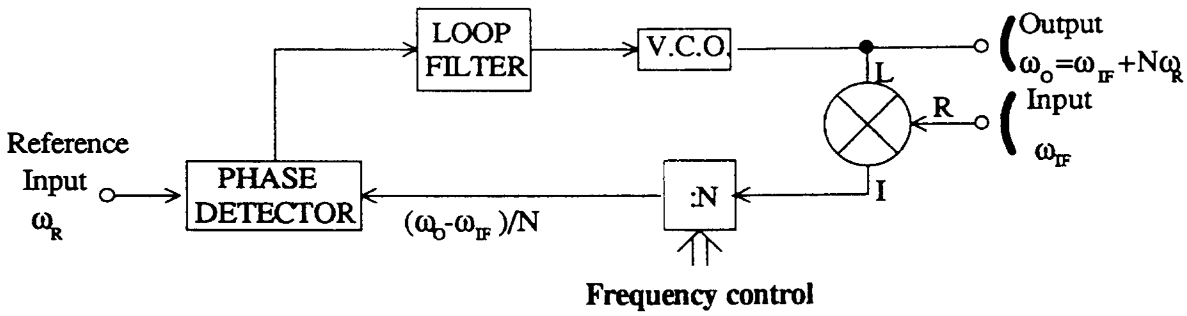


Fig. 18 Local Oscillator frequency synthesis

A combination of P.L.L. and D.D.S. techniques can also be made using the single side-band mixer principle (Section 3.2.2). As represented in Fig. 19 it makes use of the D.D.S. capability to provide simultaneously two outputs in phase quadrature. This results in an overall synthesizer having a high frequency resolution provided by D.D.S., and a high frequency range with small response time provided by the P.L.L. using a high reference frequency. The offset frequency $\omega_C/2\pi$ is not absolutely necessary, but it eases the requirements on the quadrature hybrid and it helps provide an improved output signal by a simplification of the filtering.

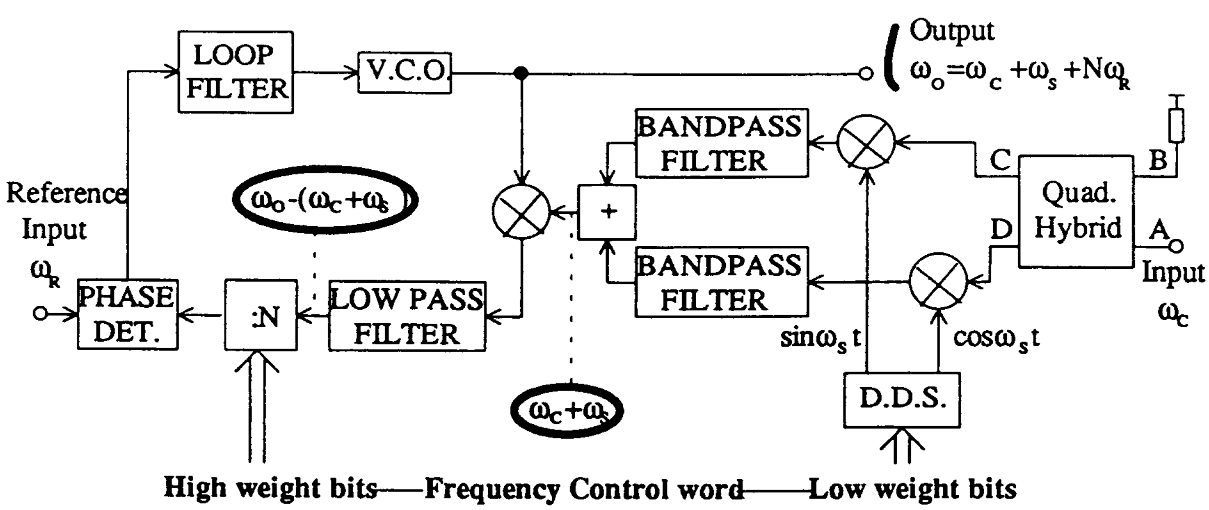


Fig. 19 High frequency / high resolution synthesizer

4. FILTERS AND PHASE-SHIFTERS.

The following section is not a course on filter design. It is only meant to give the general description of the various categories of filters. References for detailed design information are provided.

4.1 Analogue filter design method (Overview)

A filter is a device that has a prescribed transfer function (S_{12}) in the frequency domain. Any impedance matching network is in fact a filter.

In a purely passive two-port filter the input R.F. power into port 1 can be :

- reflected :
$$\frac{P_{Reflected}}{P_{Direct}} = |S_{11}|^2 \tag{18}$$

- transmitted to port 2 :
$$\frac{P_{Transmitted}}{P_{Direct}} = |S_{12}|^2 \tag{19}$$

- dissipated (thermally or radiated).

The designer's aim is generally to bring S_{12} of the real filter as close as possible to the prescribed specification. This usually means maximizing $|S_{12}|$ in the band(s) of interest and minimizing it in the band(s) to be rejected. Filter design is basically a three-step process :

i) after selection of the mathematical approximation to be used (Butterworth, Tchebyscheff, Cauer, Bessel,...) the order of the transfer function and the numerical values of the coefficients can be computed,

ii) after selection of a filter topology (active, passive, lumped or distributed elements,...) component values must be worked out,

iii) components values have to be compared to available or constructable ones. When no match is found, filter topology has to be modified, and step (ii) reiterated. When a "moderate" mismatch remains, optimization can be attempted, modifying slightly some components towards an improved feasibility.

Numerous textbooks are available, containing enough tabulated information to design all kinds of filters (Refs.[10] to [12]). However practice relies more and more nowadays upon software packages to handle all computational activities, especially optimization (TOUCHSTONE-ESYN product from EESOF company for instance).

4.2 Digital filters

Digital technology brings the benefit of predictable and reliable performance over a long time-span. Moreover it provides an easy control parameter in the form of the system clock, which allows a real time efficient and simple programming of the filter response in an accelerator, when beam frequency changes.

Analysis and description of the transfer function of digital filters is conveniently achieved using the "z-Transform" (Refs.[13] to [14]). In this representation z^{-1} corresponds to a pure delay of one sample clock period

$$z^{-1} = e^{-j\omega T} \tag{20}$$

where ω is the signal angular frequency in rad/s, and T is the clock period.

Two fundamental types of filters can be distinguished according to their response to excitation by a Dirac impulse :

- the Finite Impulse Response (F.I.R.) or transversal filter,
- the Infinite Impulse Response (I.I.R.) or recursive filter.

No feedback connection exists in an F.I.R. filter, whose structure is described in Fig. 20. The output is simply a linear combination of a finite number of previous input samples. Consequently the impulse response is also of limited duration.

The transfer function is of the form : $$F(z) = \frac{Y(z)}{X(z)} = h_0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3} + \ldots \qquad (21)$$

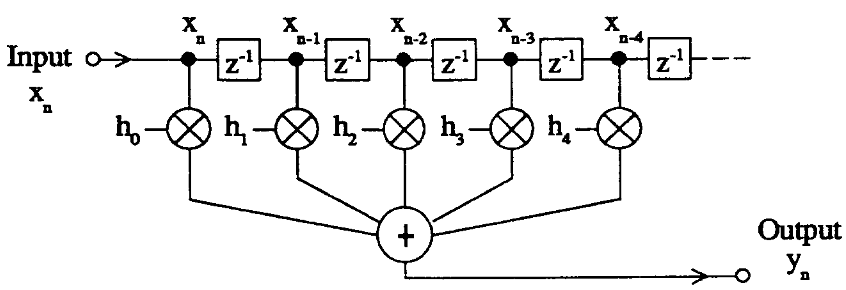


Fig. 20 Structure of an F.I.R. filter

The major advantages of this structure are :

- unconditional stability (No poles in the transfer function),
- capability to provide a linear phase (constant group delay),
- ease of design.

An I.I.R. filter makes use of a linear combination of input and output samples to generate the output signal (Fig. 21). Since loops are incorporated the stability issue deserves a detailed analysis.

The transfer function is of the form : $$F(z) = \frac{Y(z)}{X(z)} = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + \ldots}{1 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + \ldots} \qquad (22)$$

I.I.R. filters are more efficient than F.I.R. ones, with a lower number of coefficients and computing steps. However their phase is never linear with frequency (group delay is not constant).

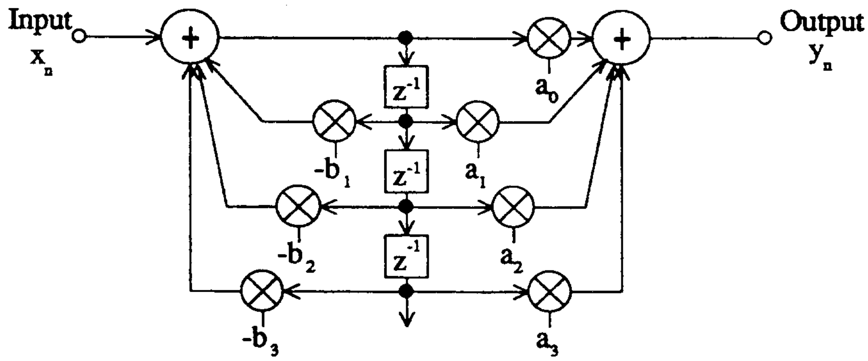


Fig. 21 Structure of an I.I.R. filter

With $b_k \neq 0$ and no other coefficient, the filter has a very simple topology. This solution has already been applied in multi-harmonic feedbacks for synchrotrons [15,16]. Provided that z^{-k} corresponds to a one-turn delay, a comb-like response is obtained, with maxima at the harmonics of the beam revolution frequency using a minimum amount of hardware (Fig. 22).

The transfer function is simply : $$F(z) = \frac{1}{1 - b_k z^{-k}} \qquad (23)$$

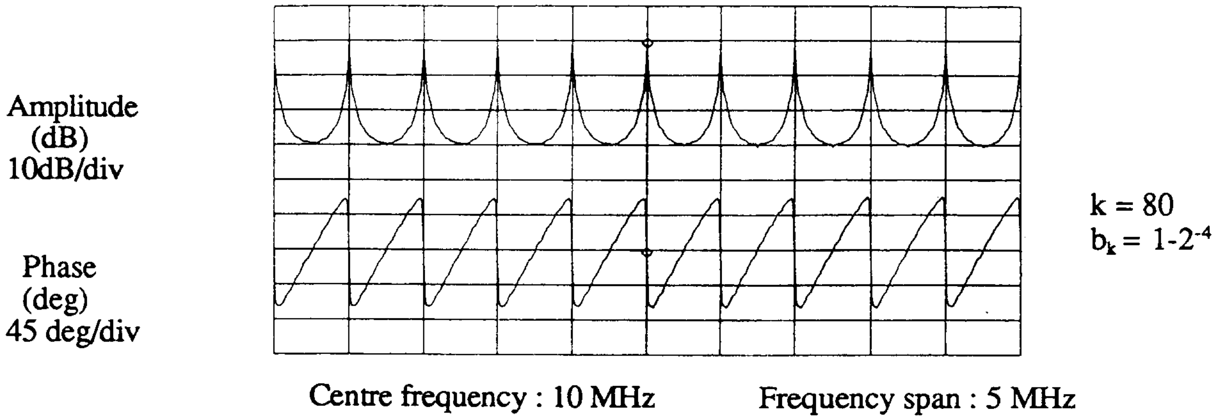


Fig. 22 Frequency response of a comb-filter

4.3 Filter with frequency transposition

Using the Superheterodyne and single side-band mixing techniques, the frequency response of a low frequency filter can be translated around any centre frequency $\omega_L/2\pi$. The typical block diagram is represented in Fig. 23. The pass-band characteristic of such a tracking filter results from filtering at a convenient and fixed intermediate frequency. This set-up is used, for instance, in feedback systems damping coupled-bunch oscillations of the beam [17]. In this case the intermediate angular frequency is at 0 rad/s (D.C.), and the I.F. filters are simple high-pass RC networks.

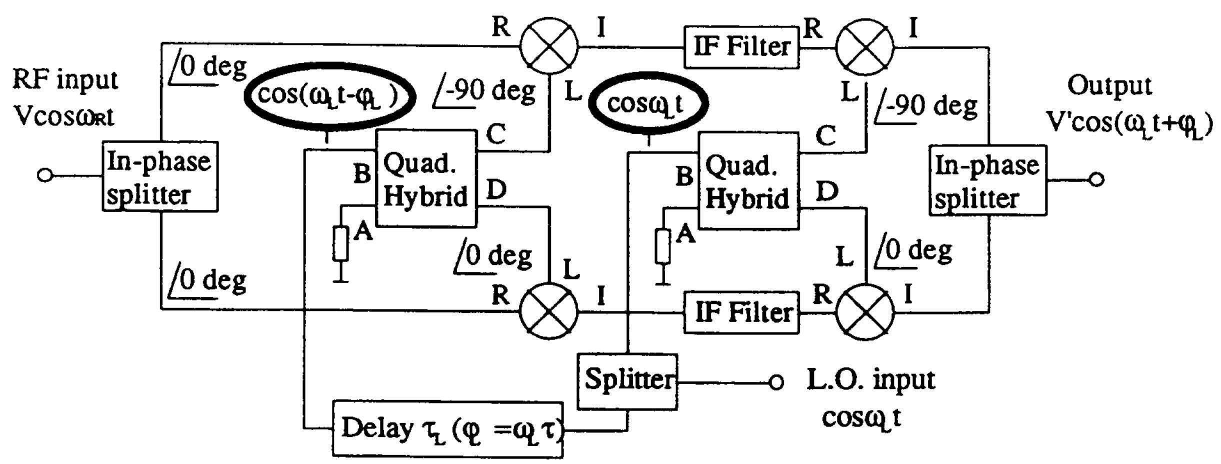


Fig. 23 Filter with frequency transposition

When the local oscillator used for the first mixing is delayed by $-\varphi_L$ with respect to the signal used for the second mixing, a phase shift $+\varphi_L$ is obtained at the output. If the first local oscillator is delayed by τ with respect to the second one, the overall filter will provide a phase advance at its centre frequency which is proportional to ω_L. This capability is often used to stabilize feedback systems, maintaining a constant phase-shift through a complete installation with a long delay while the beam energy (frequency) is changing.

4.4 Phase-shifters

A phase-shifter is a kind of filter providing control of the phase-shift through it, while keeping a constant insertion loss in the band of interest. In amplitude it can be qualified as a band-pass or even all-pass filter.

Two variants of a classical structure for an all-pass filter are shown in Fig. 24. Their transfer function is given by : $G(s) = k\left[\frac{1-\tau s}{1+\tau s}\right]$ (24)

with : $\tau = RC$ and $k = 1$ (Fig. 24 (a)) or 1/2 (Fig. 24 (b)). It corresponds to a constant amplitude response and a phase lag which is two times the one of a simple RC low-pass circuit : $\varphi = -2\arctan\tau.$ (25)

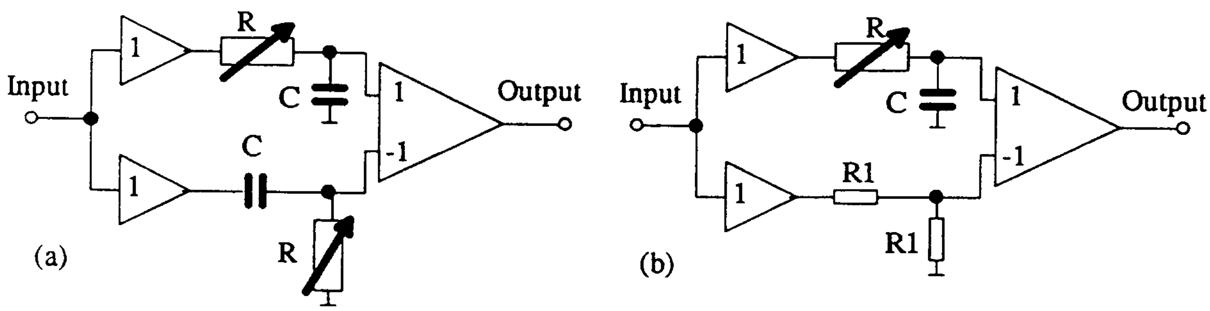


Fig. 24 All-pass filter phase-shifter

Another powerful scheme for phase-shifting is represented in Fig. 25. It can be understood as an application of the single side-band mixer principle explained in Section 3.2.2 (see Fig. 17) with $\omega_L = 0$. This provides an input which directly and linearly controls the phase-shift over a dynamic range which can be very large (many times 2π rad).

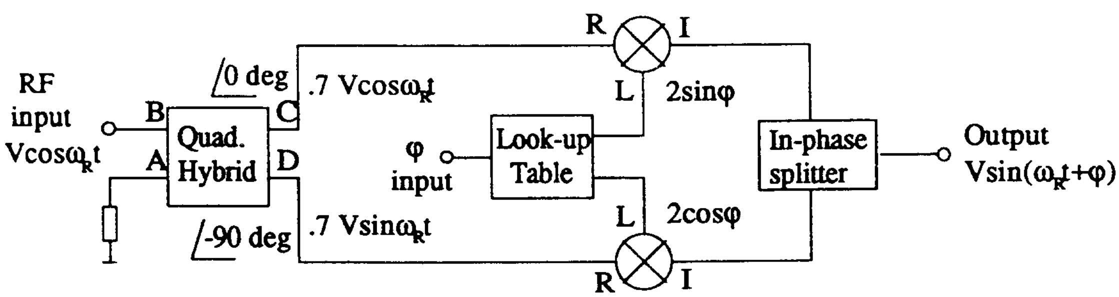


Fig. 25 Wide dynamic range phase-shifter

Above 100 MHz, a circuit using a quadrature hybrid and varicap diodes as controllable elements is sometimes used. Its principle is illustrated in Fig. 26. Its insertion loss is given by :

$$S_{12} = -j\rho \tag{26}$$

where ρ is the reflection coefficient of one varicap diode : $\rho = \dfrac{1 - Z_0 Cj\omega}{1 + Z_0 Cj\omega}$ (27)

Consequently :

$$S_{12} = -j\left[\frac{1 - jZ_0C\omega}{1 + jZ_0C\omega}\right] \Rightarrow \begin{Bmatrix} |S_{12}| = 1 \\ \operatorname{Arg}(S_{12}) = -\frac{\pi}{2} - 2\arctan(Z_0C\omega) \end{Bmatrix} \tag{28}$$

so that the circuit also behaves like an all-pass filter.

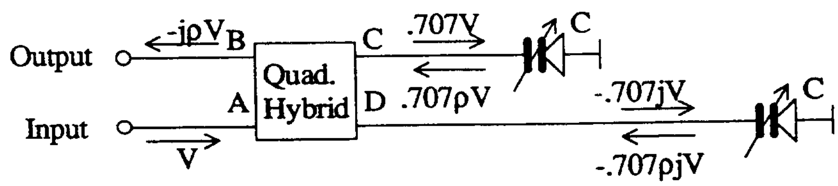


Fig. 26 Phase-shifter using quadrature hybrid and varicap diodes

4.5 Variable digital delay

The delay of transmission through a digital system equipped with an A.D.C. at the input and a D.A.C. at the output is of the form :

$$\tau_D = \tau + nT_{clock} \tag{29}$$

with τ the pure delay due to analogue elements of limited bandwidth, T_{clock} the clock period and n the number of clock periods for propagation from input to output (1/2 for the A.D.C., 1/2 for the D.A.C., plus the various registers).

τ_D is controllable with T_{clock}. But the range of delay τ_D that can be covered is limited because the bandwidth of the system is also linked to T_{clock} (bandwidth < $f_{clock}/4$).

A better solution is obtained by the application of a technique connected to the one described for the "Filter with frequency transposition" (Section 4.3). A delay τ_W is introduced between the output and input clocks of a First-In First-Out (F.I.F.O.) register (Fig. 26). When the clock frequency is f_{c0} the number of cells in the F.I.F.O. is initialized at n_0, and consequently the total delay through the system is :

$$\tau_{D0} = \tau + \frac{n_0}{f_{c0}}. \tag{30}$$

When the clock frequency has changed to f_{c1}, the number of clock periods in the delay τ_W has changed by :

$$\Delta n = (f_{c1} - f_{c0})\tau_W \tag{31}$$

and consequently the number of cells in the F.I.F.O. has changed by $-\Delta n$.

The overall delay in the system is then : $$\tau_D = \tau + \frac{n_0 - (f_{c1} - f_{c0})\tau_W}{f_{c1}} \tag{32}$$

which can also be expressed as :

$$\boxed{\tau_D = \tau + \left(n_0 + f_{c0}\tau_W\right)T_{clock} - \tau_W} \tag{33}$$

Constant delay

Constant phase-shift at any input frequency in constant ratio with f_{clock} ($f_{in}=k.f_{clock}$)

Negative delay

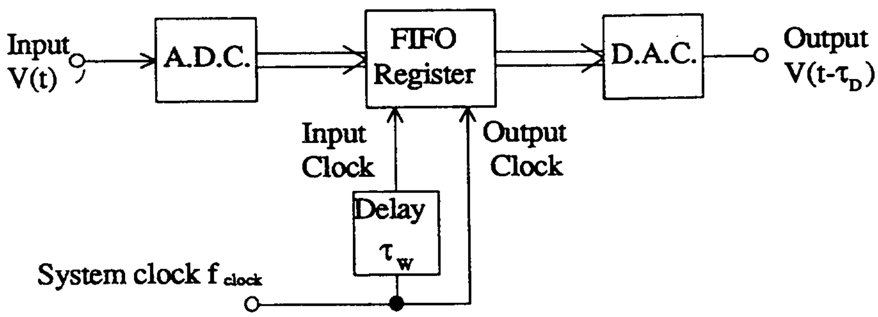


Fig. 27 Variable digital delay for automatic delay compensation

Compensation of an external delay $\tau_{ext} = \tau_W - \tau$ is possible with such a device, maintaining a constant phase-shift through a full installation when the beam energy (frequency) changes. For example, if f_{clock} is a harmonic of the revolution frequency in a synchrotron, the automatic delay compensation device will guarantee a constant phase-shift simultaneously at all revolution harmonics inside the system bandwidth [16].

5. MODULATION

5.1 Definition and vector representation

Modulation is needed to transmit information on an R.F. carrier. In fact modulation is involved every time a signal departs from a pure sinusoidal function of time.

The time dependance of a carrier sine-wave at ω_C, modulated in amplitude (a(t)) and phase (p(t)) is expressed as :

$$x(t) = \mathrm{Re}\left\{\hat{X}\left(1 + a(t)\right)e^{j(\omega_C t + \varphi(t))}\right\}. \tag{34}$$

For purely sinusoidal modulations :

$$x(t) = \mathrm{Re}\left\{\hat{X}\left(1 + \hat{a}\cos\omega_{AM}t\right)e^{j(\omega_C t + \hat{\varphi}\sin\omega_{PM}t)}\right\} \tag{35}$$

and for small amplitudes of modulation a convenient approximation is :

$$x(t) \approx \mathrm{Re}\left\{\hat{X}(1+\hat{a}\cos\omega_{AM}t)(1+j\hat{\varphi}\sin\omega_{PM}t)e^{j\omega_C t}\right\} \tag{36}$$

which can be reorganized into :

$$\boxed{x(t) \approx \mathrm{Re}\left\{\hat{X}\left[\underbrace{e^{j\omega_C t}}_{\text{Carrier}} + \underbrace{\frac{\hat{a}}{2}\left(e^{j(\omega_C+\omega_{AM})t} + e^{j(\omega_C-\omega_{AM})t}\right)}_{\text{A.M. side-bands}} + \underbrace{\frac{\hat{\varphi}}{2}\left(e^{j(\omega_C+\omega_{PM})t} - e^{j(\omega_C-\omega_{PM})t}\right)}_{\text{P.M. side-bands}}\right]\right\}} \tag{37}$$

Equation (37) suggests a graphics representation, where each term of the form $e^{j\omega t}$ corresponds to a vector. In a reference frame rotating at the carrier angular frequency ω_C rad/s the picture for amplitude modulation alone is given in Fig. 28 (a), together with its frequency spectrum. Fig. 28 (b) illustrates the case of phase modulation.

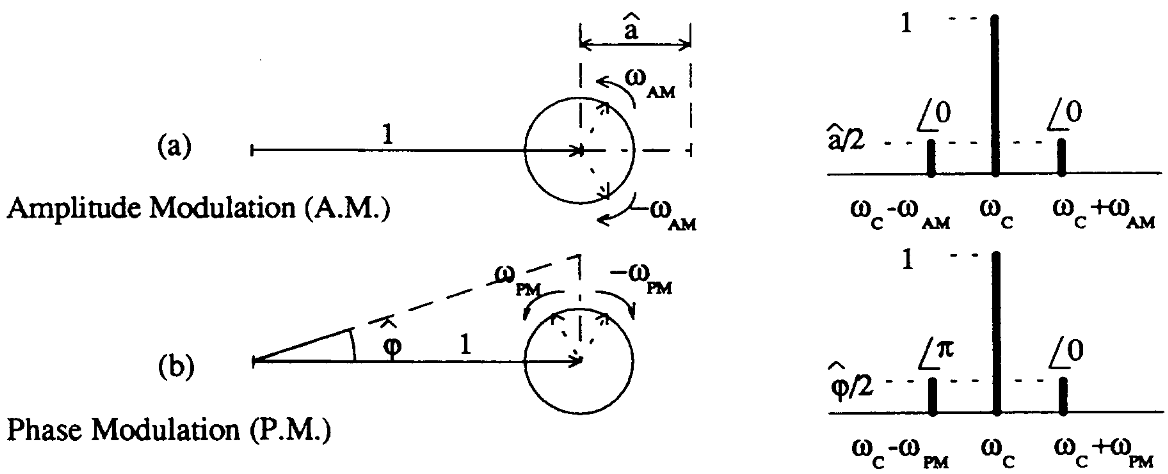


Fig. 28 Graphics representation of modulation

5.2 Transmission through a linear system

Transmission of modulation can be derived from the system transfer function $H(j\omega)$. A set of four modulation transfer functions have to be worked out :

i) $G_{aa}(j\omega)$ for amplitude modulation to amplitude modulation

ii) $G_{pp}(j\omega)$ for phase to phase modulation

iii) $G_{ap}(j\omega)$ for amplitude to phase modulation

iv) $G_{pa}(j\omega)$ for phase to amplitude modulation.

$G_{pa}(j\omega) = G_{ap}(j\omega) = 0$ is an exception, which is observed for instance when $H(j\omega)$ is a pure delay.

For small amplitudes of modulation of the input signal $x(t)$, Equation (37) can be used, and the output signal $y(t)$ is :

$$y(t) = \hat{X} \cdot \mathrm{Re}\left\{\begin{bmatrix} H(j\omega_C)e^{j\omega_c t} + \\ +\frac{\hat{a}}{2}\left[H(j(\omega_C + \omega_{AM}))e^{j(\omega_C+\omega_{AM})t} + H(j(\omega_C - \omega_{AM}))e^{j(\omega_C-\omega_{AM})t}\right] \\ +\frac{\hat{\varphi}}{2}\left[H(j(\omega_C + \omega_{PM}))e^{j(\omega_C+\omega_{PM})t} - H(j(\omega_C - \omega_{PM}))e^{j(\omega_C-\omega_{PM})t}\right] \end{bmatrix}\right\} \tag{38}$$

which can be written :

$$y(t) = \hat{X} \cdot \mathrm{Re}\left\{H(j\omega_C)e^{j\omega_c t}\begin{bmatrix} 1 \\ +\frac{\hat{a}}{2}\left[\frac{H(j(\omega_C + \omega_{AM}))}{H(j(\omega_C))}e^{j\omega_{AM}t} + \frac{H(j(\omega_C - \omega_{AM}))}{H(j(\omega_C))}e^{-j\omega_{AM}t}\right] \\ +\frac{\hat{\varphi}}{2}\left[\frac{H(j(\omega_C + \omega_{PM}))}{H(j(\omega_C))}e^{j\omega_{PM}t} - \frac{H(j(\omega_C - \omega_{PM}))}{H(j(\omega_C))}e^{-j\omega_{PM}t}\right] \end{bmatrix}\right\} \tag{39}$$

With the definitions :

$$G_S(j\omega) = \frac{1}{2}\left[\frac{H(j(\omega + \omega_C))}{H(j\omega_C)} + \frac{H(j(\omega - \omega_C))}{H(-j\omega_C)}\right] \tag{40}$$

$$G_C(j\omega) = \frac{j}{2}\left[\frac{H(j(\omega + \omega_C))}{H(j\omega_C)} - \frac{H(j(\omega - \omega_C))}{H(-j\omega_C)}\right] \tag{41}$$

we get :

$$y(t) = \hat{X} \cdot \mathrm{Re}\left\{H(j\omega_C)e^{j\omega_C t}\begin{bmatrix} 1 \\ +\frac{\hat{a}}{2}\left[G_S(j\omega_{AM})e^{j\omega_{AM}t} + G_S^*(j\omega_{AM})e^{-j\omega_{AM}t}\right] \\ -j\frac{\hat{a}}{2}\left[G_C(j\omega_{AM})e^{j\omega_{AM}t} + G_C^*(j\omega_{AM})e^{-j\omega_{AM}t}\right] \\ +\frac{\hat{\varphi}}{2}\left[G_S(j\omega_{PM})e^{j\omega_{PM}t} - G_S^*(j\omega_{PM})e^{-j\omega_{PM}t}\right] \\ -j\frac{\hat{\varphi}}{2}\left[G_C(j\omega_{PM})e^{j\omega_{PM}t} - G_C^*(j\omega_{PM})e^{-j\omega_{PM}t}\right] \end{bmatrix}\right\} \tag{42}$$

$$y(t) = \hat{X} \cdot \mathrm{Re}\left\{ H(j\omega_C) e^{j\omega_C t} \begin{bmatrix} 1 \\ +\hat{a}\left[\mathrm{Re}\left\{G_S(j\omega_{AM}) e^{j\omega_{AM} t}\right\}\right] \\ +\hat{\varphi}\left[\mathrm{Im}\left\{G_C(j\omega_{PM}) e^{j\omega_{PM} t}\right\}\right] \\ -j\hat{a}\left[\mathrm{Re}\left\{G_C(j\omega_{AM}) e^{j\omega_{AM} t}\right\}\right] \\ +j\hat{\varphi}\left[\mathrm{Im}\left\{G_S(j\omega_{PM}) e^{j\omega_{PM} t}\right\}\right] \end{bmatrix} \right\} \tag{43}$$

and consequently the modulations at the output are :

$$\begin{aligned} a_Y(t) &= \hat{a} \cdot \mathrm{Re}\left\{G_S(j\omega_{AM}) e^{j\omega_{AM} t}\right\} + \hat{\varphi} \cdot \mathrm{Im}\left\{G_C(j\omega_{PM}) e^{j\omega_{PM} t}\right\} \\ \varphi_Y(t) &= \hat{\varphi} \cdot \mathrm{Im}\left\{G_S(j\omega_{PM}) e^{j\omega_{PM} t}\right\} - \hat{a} \cdot \mathrm{Re}\left\{G_C(j\omega_{AM}) e^{j\omega_{AM} t}\right\}. \end{aligned} \tag{44}$$

Owing to the definition of the input modulation, we can then write :

$$\boxed{\begin{aligned} G_{aa}(j\omega) = G_{pp}(j\omega) = G_S(j\omega) &= \frac{1}{2}\left[\frac{H(j(\omega+\omega_C))}{H(j\omega_C)} + \frac{H(j(\omega-\omega_C))}{H(-j\omega_C)}\right] \\ G_{pa}(j\omega) = -G_{ap}(j\omega) = G_C(j\omega) &= \frac{j}{2}\left[\frac{H(j(\omega+\omega_C))}{H(j\omega_C)} - \frac{H(j(\omega-\omega_C))}{H(-j\omega_C)}\right] \end{aligned}} \tag{45}$$

Application: transmission of modulation through a resonator (typical case of an R.F. cavity)

The resonator impedance is given by :

$$Z(s) = \frac{2\sigma R s}{s^2 + 2\sigma s + \omega_R^2} \tag{46}$$

with R the resonator shunt resistance (Ω), ω_R the resonance angular frequency (rad/s), σ $\left(\sigma = \frac{\omega_R}{2Q}\right)$ the damping rate (s^{-1}), and Q the quality factor of the resonator.

When driven by a current generator with a carrier frequency ω_C, the transfer functions for modulation are then :

$$\begin{aligned} G_{aa} = G_{pp} &= \frac{\sigma^2(1+\tan^2\varphi_Z) + \sigma s}{s^2 + 2\sigma s + \sigma^2(1+\tan^2\varphi_Z)} \\ G_{pa} = -G_{ap} &= \frac{\sigma \tan\varphi_Z s}{s^2 + 2\sigma s + \sigma^2(1+\tan^2\varphi_Z)} \end{aligned} \tag{47}$$

defining φ_Z by : $\quad \sigma \tan \varphi_Z = \omega_R - \omega_C.$ (48)

For a carrier centred at the resonance frequency, the system behaves like a first-order low-pass filter for both types of modulation, with a 3 dB cut-off at σ rad/s. In such a case, no coupling exists between modulations. The transfer functions are :

$$
\begin{aligned}
G_{aa} &= G_{pp} = \frac{\sigma}{s+\sigma} \\
G_{pa} &= -G_{ap} = 0
\end{aligned}
\tag{49}
$$

5.3 Techniques of modulation and demodulation

5.3.1 Phase modulation

The various techniques applied for phase modulation have been described in the preceding sections :

- frequency modulation of a source (V.C.O., D.D.S.),
- perturbation of a P.L.L. (sect. 2.1.3),
- controlled all-pass filters (sect. 4.4).

5.3.2 Amplitude modulation

Analogue (section 3.1) or digital multipliers (section 2.2.3) are the components of choice for the control of amplitude at low power levels. At high frequencies (> 100 MHz) variable attenuators are sometimes employed. PIN diodes whose R.F. resistance is controlled by their bias current are used as variable elements. Figure 29 shows a design using a quadrature hybrid, similar to the circuit described for phase-shifting in section 4.4.

Its insertion loss is :

$$
S_{12} = \frac{(Z_C//R) - Z_0}{(Z_C//R) + Z_0} \tag{50}
$$

S_{12} is always real : attenuation varies with R, but phase-shift stays constant,

S_{12} is minimum (large attenuation) when $R >> Z_C$,

S_{12} is maximum (low insertion loss) when $R \sim 0\ \Omega$ (diodes in short-circuit).

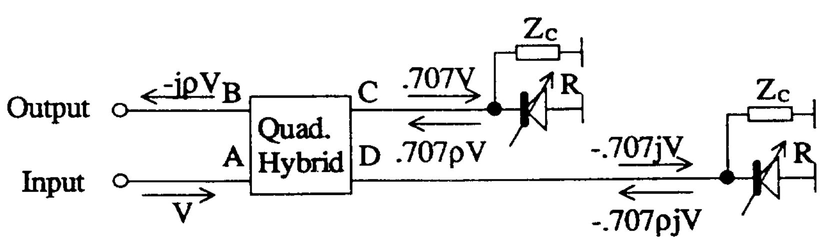


Fig. 29 Controlled attenuator

5.3.3 Phase demodulation

The miscellaneous types of phase detectors have been detailed in section 2.1.2.

5.3.4 Amplitude demodulation

The simplest amplitude detector is the peak envelope detector, represented in Fig. 30. Although very easy to implement, it is plagued by a limited linear dynamic range (~ 30 dB) due to the minimum forward voltage drop in the diode (~0.3 V for a Schottky unit).

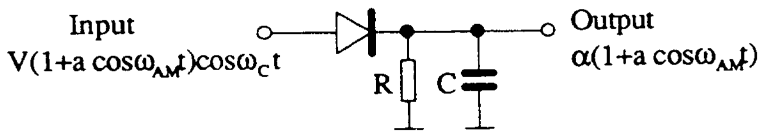


Fig. 30 Peak envelope detector

A wider dynamic range (~ 50 dB) is obtained with the synchronous detector shown in Fig. 31. Full-wave rectification of the carrier signal is obtained by multiplication of the input signal with a squared version of itself.

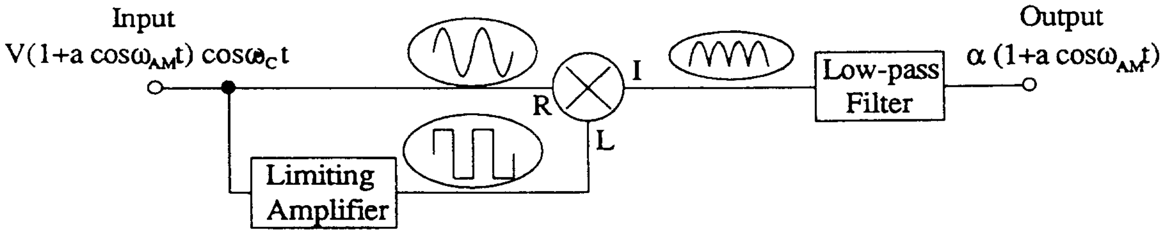


Fig. 31 Synchronous amplitude detector (full-wave rectifier)

The widest dynamic range (~ 70 dB) requires quasi-logarithmic detection as approximated by successive detection. Figure 32 shows a typical circuit (from Plessey documentation) together with its performance.

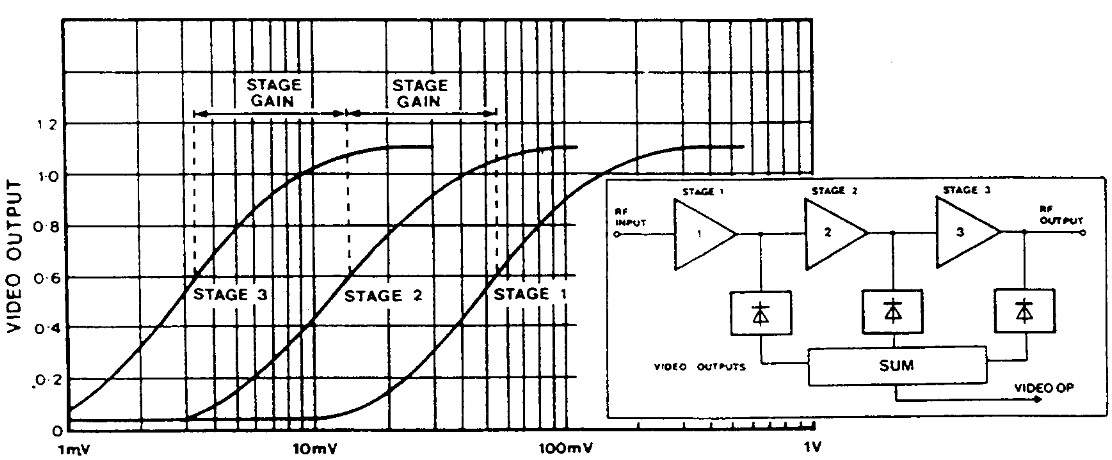


Fig. 32 Quasi-logarithmic detector (successive detection)

5.3.5 Vector modulation/demodulation

Any modulated signal with a carrier angular frequency ω_C can be described by :

$$s(t) = a(t)\sin(\omega_C t + \varphi(t)) \tag{51}$$

but also by :

$$s(t) = \underbrace{i(t)}_{\text{In-phase component "I"}}\sin\omega_C t + \underbrace{q(t)}_{\text{Quadrature component "Q"}}\cos\omega_C t \tag{52}$$

The I and Q components can be used as abscissa and ordinate in a rectangular coordinates system, giving a display equivalent to Fig. 28. Any modulation has a characteristic representation in the I-Q plane, and all information relative to the modulation is available. Figure 33 shows the elementary case of C.W. (no modulation), pure A.M., and pure P.M.. Some general purpose commercial instruments begin to appear on the market (HP8980 from Hewlett-Packard for example [18]) which provide a new insight into the modulation domain.

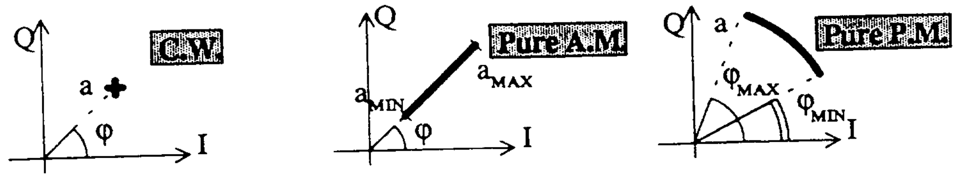


Fig. 33 I-Q displays for some elementary modulations

Digital technology is likely to help designers to obtain the full benefit of this technique, providing easy to use components for processing the I-Q data. Plessey is, for instance, commercializing a "Pythagoras Processor" (PPSP1633 [19]) which accepts digital I-Q data at 15 MHz, and gives amplitude and phase (digital words) at its output.

Practically, the I and Q channels are obtained using a vector demodulator, with the methods reported in preceding sections. The demodulator described in Fig. 34 (a) is precisely the left half of the "filter with frequency transposition" in Fig. 23.

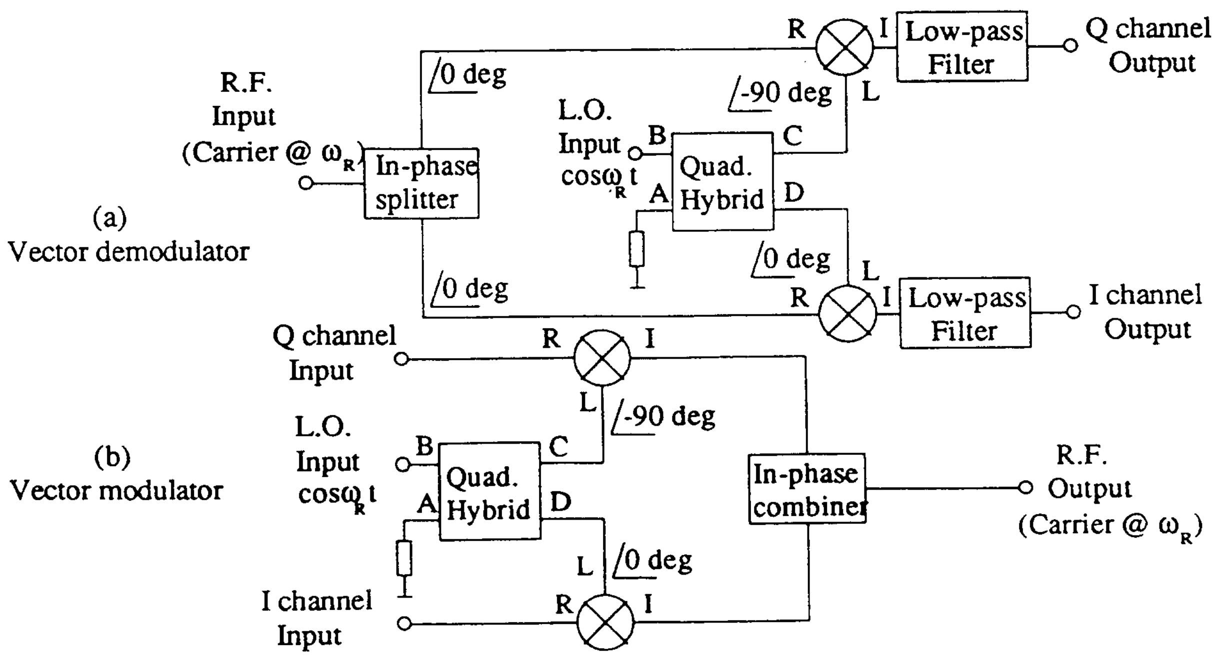


Fig. 34 Vector modulator/demodulator

A vector modulator, which generates the modulated carrier from the I-Q signals, is shown in Fig. 34(b), and is exactly the circuit used in the right half of Fig. 23.

5.3.6 "Δ/Σ" demodulation

The physical parameter of interest is often related to the amplitude ratio Δ/Σ of two R.F. signals (direction of echo in mono-pulse radar, beam position in a P.U.). Figure 35 shows an efficient means to derive this information, using only phase measurement hardware. The vector corresponding to the Δ signal is added in phase quadrature to the Σ signal to generate S. The phase angle θ between Σ and Σ is related to Δ/Σ by :

$$\theta = \mathrm{Arg}\left(\frac{\vec{S}}{\vec{\Sigma}}\right) = \arctan\left(\frac{\Delta}{\Sigma}\right). \tag{53}$$

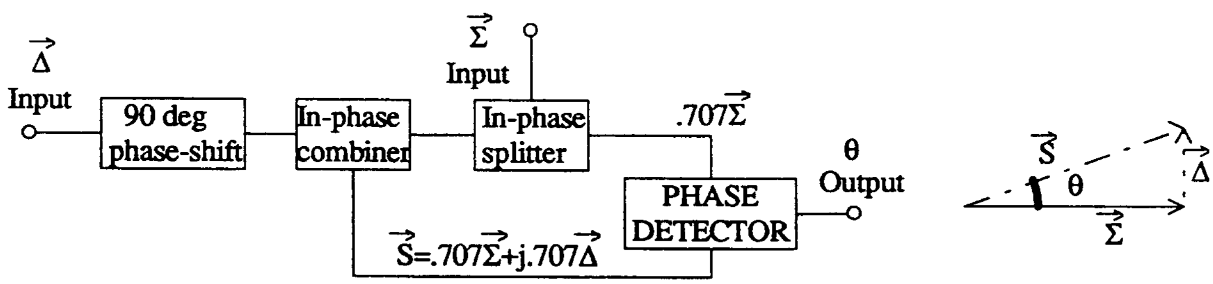


Fig. 35 "Δ/Σ" demodulator

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